

The goal of the following exercises is to be able to solve systems of ordinary differential equations without knowledge of the chemical and engineering context.

### Exercise 1: gas-phase parallel reactions in a tubular reactor

Here's a problem to solve using Python! We have a reagent A reacting in 3 parallel chemical reactions to form a desired product B and two unwanted products X and Y.

The differential equations to be solved to model this reaction in a tubular reactor are:

$$\frac{dF_A}{dV} = -k_1 C_A - k_2 - 2k_3 C_A^2$$

$$\frac{dF_B}{dV} = k_1 C_A$$

$$\frac{dF_X}{dV} = k_2$$

$$\frac{dF_Y}{dV} = 2k_3 C_A^2$$

$$C_A = F_A/2$$

With constants  $k_1$ ,  $k_2$  and  $k_3$  of 0.0012, 0.00011 and 0.0041 respectively. The values of  $F_A$ ,  $F_B$ ,  $F_X$  and  $F_Y$  for  $V=0$  are 0.92 mol/s, 0 mol/L, 0 mol/s and 0 mol/s, respectively.

What are  $V$  (in liters) and  $F_B$  (in mol/s) when  $F_A$  is half its initial value (the conversion is 50 %)?

### Exercise 2 : fed-batch reaction with a chemical equilibrium

We wish to solve the two differential equations below for  $t = 5000$  s with Python.

$$R = k_1 \cdot C_A C_B - k_2 \cdot C_C C_D$$

$$\frac{dC_A}{dt} = -R - \frac{\dot{V}}{V} \cdot C_A$$

$$\frac{dV}{dt} = \dot{V}$$

$$\frac{dC_B}{dt} = -R + \frac{\dot{V}}{V} \cdot (C_{BF} - C_B)$$

$$\frac{dC_C}{dt} = R - \frac{\dot{V}}{V} \cdot C_C$$

$$\frac{dC_D}{dt} = R - \frac{\dot{V}}{V} \cdot C_D$$

With:

$$k_1 = 9.00 \cdot 10^{-5}, k_2 = 8.33 \cdot 10^{-5}$$

$$\dot{V} = 0.05 \text{ for } t \leq 4000 \text{ s and } \dot{V} = 0 \text{ otherwise}$$

$$C_{BF} = 10.93 \text{ mol/L}$$

The initial conditions are:

$$C_{A0} = 7.72 \text{ mol/L}$$

$$V_0 = 200 \text{ L}$$

$$C_{B0} = 0 \text{ mol/L}$$

$$C_{C0} = 0 \text{ mol/L}$$

$$C_{D0} = 0 \text{ mol/L}$$

What will be the concentration B when we stop feeding at  $t = 4000 \text{ s}$  ? What will be the concentrations of A, B, C and D after  $5000 \text{ s}$  ?

### Exercise 3: tubular reactor with exothermic reaction

You are asked to solve the problem described by the following ordinary differential equations using the Python programming language:

$$\frac{dF_A}{dV} = R_A \text{ avec } R_A = -r_1 + r_2 \quad r_1 = k \cdot C_A \quad r_2 = \left(\frac{k}{K}\right) \cdot C_B$$

$$\frac{dF_B}{dV} = R_B \text{ avec } R_B = r_1 - r_2 \quad C_A = F_A/Vdot0 \quad C_B = F_B/Vdot0$$

$$\frac{dT}{dV} = \frac{(-\Delta H_R) \cdot r_1 - (-\Delta H_R) \cdot r_2 + Ua \cdot (T_j - T)}{F_A \cdot c_{pA} + F_B \cdot c_{pB} + F_{I0} \cdot c_{pI}} \quad k = 31.12 \cdot \exp\left[\frac{E_a}{8.314} \left(\frac{1}{360} - \frac{1}{T}\right)\right]$$

$$K = 3.031 \cdot \exp\left[\frac{\Delta H_R}{8.314} \left(\frac{1}{333} - \frac{1}{T}\right)\right]$$

The constant values are

$$C_{A0} = 1.86 \text{ kmol/m}^3, F_{A0} = 14.67 \text{ kmol/h}, F_{B0} = 0 \text{ kmol/h}, F_{I0} = 1.63 \text{ kmol/h}, E_a = 65700 \text{ J/mol}, \\ T_0 = 305 \text{ K}, T_j = 315 \text{ K}, \Delta H_R = -34500 \text{ kJ/kmol}, c_{pA} = c_{pB} = 141 \text{ kJ/kmol/K}, c_{pI} = 161 \\ \text{ kJ/kmol/K}, Ua = 5000 \text{ kJ/m}^3\text{/h/K}, Vdot0 = F_{A0}/C_{A0} \text{ m}^3\text{/h}$$

The values  $F_A$ ,  $F_B$  and  $T$  for  $V = 0$  are the constants  $F_{A0}$ ,  $F_{B0}$  et  $T_0$  given above. Draw profiles of  $F_A$ ,  $F_B$  and  $T$  as a function of  $V$  and find the max value of  $T$  in the reactor.